# Disorder in order: simulating a random scattering potential with a randomless cold atom system

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Polarons in the 21st Century
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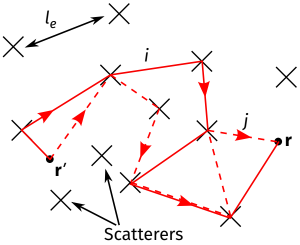
# Introduction — disorder in quantum physics

Disorder is at the heart of several quantum phenomena:

- Electronic transport at low temperatures.
- Scattering of light by disorder (speckle pattern, back-scattering).
- Anderson localization via wave interference effects [Anderson, PR ’58].

“Simple” image: density $n(\mathbf{r}, t)$ expressed as a sum over scattering paths.

$$n(\mathbf{r}, t) = \Big| \sum_{\text{path } i} A_i \Big|^2 = \underbrace{\sum_{\text{path } i} |A_i|^2}_{\text{classical}} + \underbrace{\sum_{\text{path } i \neq \text{path } j} A_i A_j^*}_{\text{quantum}}.$$




$l_e$: elastic mean-free path.

Classical contribution: diffusion.

Quantum corrections (path interference) that survive disorder average can reduce diffusion.

→ localization.

1 and 2$d$: always localized, 3$d$: Anderson transition.

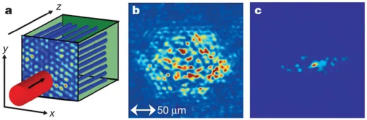
# Observations of Anderson localization

Initial motivation: spin transport in doped semiconductors [Feher and Gere, PR '59].

Issue: difficult to separate disorder and many-body effects...

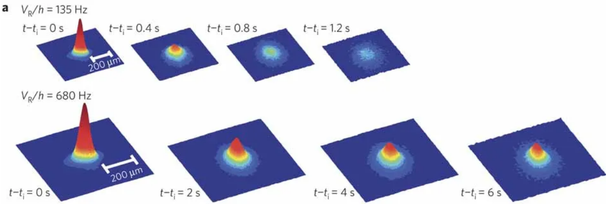
Wave phenomenon: observation with

- microwaves [Dalichaouch et al., Nature '91];
- light waves [Schwartz et al., Nature '07].




Light propagating through a photonic lattice without (left) and with (right) disorder.

Experiments in atomic matter waves [Billy et al., Nature '08; Jendrzejewski et al., Nature '12].




Expansion of a 3*d* BEC in presence of a weakly- (top) or strongly- (top) disordered speckle potential.

# A simple model of disorder

Edwards model: describe e.g. magnetic impurities in a metal, [Edwards, PM '58]

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + \underbrace{V(\hat{\mathbf{r}})}_{\text{random potential}}, \qquad V(\hat{\mathbf{r}}) = \sum_{i=1}^{N} v(\hat{\mathbf{r}} - \mathbf{r}_i).$$

- $\mathbf{r}_i$: position of $N$ scattering impurities, chosen randomly (e.g. uniform).
- $v(\mathbf{r})$: scattering potential, e.g. $v(\mathbf{r}) = g\delta(\mathbf{r})$ (random Kronig-Penney model).

## Disorder average of observables

$$\overline{\langle \hat{n}(\mathbf{r}, t) \rangle} \propto \int d\mathbf{r}_1 \cdots d\mathbf{r}_N \, \langle \psi^{\{\mathbf{r}_i\}}(t) | \hat{n}(\mathbf{r}) | \psi^{\{\mathbf{r}_i\}}(t) \rangle,$$

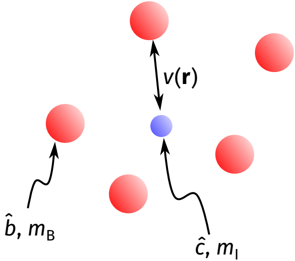
$|\psi^{\{\mathbf{r}_i\}}(t)\rangle = e^{-i\hat{H}^{\{\mathbf{r}_i\}} t} |\psi_0\rangle$ : state evolved for given disorder configuration $\{\mathbf{r}_i\}$.

# Bose polaron model

## Fermionic impurity immersed in a bosonic bath

$$\hat{H} = \underbrace{\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}} \hat{c}_{\mathbf{k}}}_{\hat{c}:\ \text{impurity (free particle)}} + \underbrace{\sum_{\mathbf{k}} \omega_{\mathbf{k}} \hat{b}^\dagger_{\mathbf{k}} \hat{b}_{\mathbf{k}}}_{\hat{b}:\ \text{bath with no interactions}} + \underbrace{\int_{\mathbf{r},\mathbf{r}'} \hat{c}^\dagger_{\mathbf{r}} \hat{c}_{\mathbf{r}} v(\mathbf{r}-\mathbf{r}') \hat{b}^\dagger_{\mathbf{r}'} \hat{b}_{\mathbf{r}'}}_{\text{interspecies interaction}} .$$

- $\hat{c}$: single impurity → wavefunction $\psi(\mathbf{r})$.
- $\hat{b}$: free boson bath, prepared initially in $|\mathrm{BEC}\rangle \propto (\hat{b}^\dagger_{\mathbf{k}=0})^N |0\rangle$.
- $v(\mathbf{r})$: density-density interaction.




Recent observations:

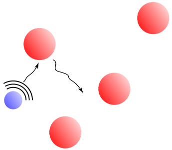
$^{39}$K in $^{39}$K [Jørgensen et al., PRL '16], $^{40}$K in $^{87}$Rb [Hu et al., PRL '16], $^{40}$K in $^{23}$Na [Yu et al., arXiv '19].

Heavy bath limit: $m_B \to \infty$, $\omega_{\mathbf{k}} \to 0$.

**Step 1**: assume that at $t = 0$, $|\Xi_0\rangle = |\psi\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle$.

[Grover and Fischer, JSM '14]

Then $\begin{cases} m_B = \infty \to \text{bosons remain in } |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle, \\ \text{impurity feels potential of scatterers at } \{\mathbf{r}_i\}. \end{cases}$



Massive bosons

≃ fixed scatterers.

i.e. $|\Xi(t)\rangle = |\psi^{\{\mathbf{r}_i\}}(t)\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle$

where $|\psi^{\{\mathbf{r}_i\}}(t)\rangle$ is evolved with $\underbrace{H^{\{\mathbf{r}_i\}} = \frac{\hat{\mathbf{p}}^2}{2m_I} + \sum_{i=1}^{N} v(\hat{\mathbf{r}} - \mathbf{r}_i).}_{\text{Edwards Hamiltonian}}$

# Mapping to the Edwards model II

(Because $|\mathrm{BEC}\rangle \propto (\hat{b}^\dagger_{\mathbf{k}=0})^N|0\rangle$!)

**Step 2**: if system initially prepared in $|\Xi_0\rangle = |\psi_0\rangle \otimes |\mathrm{BEC}\rangle \propto \int_{\{\mathbf{r}_i\}} |\psi\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle$,

$$|\Xi(t)\rangle \propto \int_{\{\mathbf{r}_i\}} |\psi^{\{\mathbf{r}_i\}}(t)\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle.$$

For any observable $\hat{O}$ of the impurity (e.g. $\hat{\mathbf{r}}$, $\hat{\mathbf{r}}^2$, $\hat{n}(\mathbf{r})$),

$$\boxed{\underbrace{\langle \Xi(t)|\hat{O}|\Xi(t)\rangle}_{\text{many-body}} = \int_{\{\mathbf{r}_i\}} \underbrace{\langle \psi^{\{\mathbf{r}_i\}}(t)|\hat{O}|\psi^{\{\mathbf{r}_i\}}(t)\rangle}_{\text{single particle}}.}$$

Many-body measurement ≡ disorder average for the Edwards model.

# Properties of the mapping

Disorder-free, many-body bose polaron model

$\Longleftrightarrow$

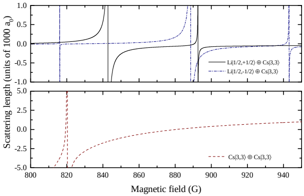
disordered, single-particle Edwards model.

Generality of the mapping?

- $v(\mathbf{r})$ can be any potential $\rightarrow$ probe universality of disorder.
- Bath prepared in state $|\Phi\rangle \rightarrow \{\mathbf{r}_i\}$ sampled with $p(\{\mathbf{r}_i\}) = |\langle\{\mathbf{r}_i\}|\Phi\rangle|^2$.
  More complicated disorders can be explored: $|\Phi\rangle$ Fermi sea?
- Remains valid with several impurities: metallic transport, many-body localization?

Limit: in real life $m_B < \infty$. Hopefully not too bad at short times.

# Leads for observation?




Candidate for large mass imbalance: [Häfner et al., PRA '17]

$^6$Li in $^{133}$Cs, $m_B/m_l = 22.1$.

Cs-Li $B \simeq 889$G resonance, $a_{\text{Cs-Cs}} = 150 a_0$.

Could 3*d* Anderson localization be investigated?

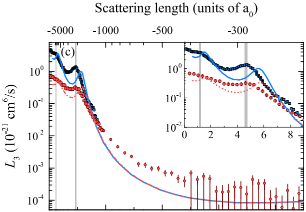
From disordered metals: transition at $k_F l_e \sim 1$; mean free path $l_e \simeq 1/(n_{\text{Cs}} a^2_{\text{Li-Cs}})$.

Three body losses: $\partial_t N_{\text{Li}}/N_{\text{Li}} \sim -L_3(a_{\text{Li-Cs}}) n^2_{\text{Cs}}$.

Possible observation: $a_{\text{Li-Cs}}$ "small" and

- Fermi sea of $^6$Li;
- few impurities at "small" **k**.

E.g.: Raman spectroscopy, [Shkedrov et al., arXiv '19].



# Variational method

Our goal: simple study of the polaron model via time-dependent variational method.

Tool: Lee–Low–Pines transform [Lee et al., PR '53].

Idea: move to the reference frame of the impurity via $\hat{S} = \exp\Big(\mathrm{i}\,\overbrace{\hat{\mathbf{r}}}^{\text{impurity}} \cdot \underbrace{\sum_{\mathbf{k}} \mathbf{k}\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}}}_{\text{boson momentum}}\Big)$.

In the new frame $\hat{H}^{\mathrm{LLP}} = \hat{S}^\dagger \hat{H} \hat{S}$, the impurity momentum **p** is conserved.

Extensively used: [Devreese and Alexandrov, RPP '09; Shashi et al., PRA '14; Schadilova et al., PRL '16...].

$$|\Xi_0\rangle = |\mathbf{p}\rangle \otimes |\mathrm{BEC}\rangle \implies |\Xi(t)\rangle = |\mathbf{p}\rangle \otimes |\mathrm{BEC}_{\mathbf{p}}(t)\rangle,$$

the BEC is evolved under a **p**-dependent Hamiltonian $\hat{H}^{\mathbf{p}}$.

$$\hat{H}^{\mathbf{p}} = \frac{(\mathbf{p} - \sum_k \mathbf{k}\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}})^2}{2m_I} + \sum_{\mathbf{k}} \omega_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}} + g\sum_{\mathbf{k}\mathbf{k}'} \hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}'}.$$

Coherent state approximation: $|\mathrm{BEC}_{\mathbf{p}}(t)\rangle \propto \exp\Big(\sum_{\mathbf{k}} \beta^{\mathbf{p}}_{\mathbf{k}}(t)\hat{b}^\dagger_{\mathbf{k}}\Big)|0\rangle$,

$\beta^{\mathbf{p}}_{\mathbf{k}}(t)$ found by variational method: minimize $(\mathrm{i}\partial_t - \hat{H}^{\mathbf{p}})|\mathrm{BEC}_{\mathbf{p}}(t)\rangle$ wrt $\partial_t \beta^{\mathbf{p}}_{\mathbf{k}}(t)$.

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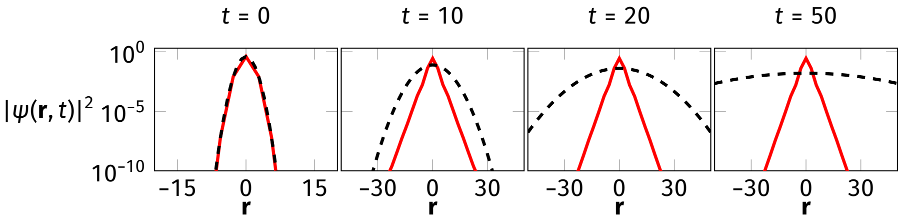
# Results: spread of the wavepacket

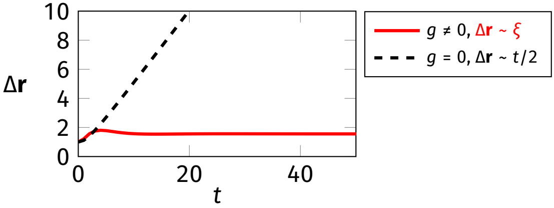
Localization is observed on the spread of a wavepacket: $[\mathbf{r} \to \mathbf{r}/\sigma, t \to t/(2m_1\sigma^2/\hbar)]$

If $\boxed{\psi(\mathbf{r}, t=0) \propto e^{-\mathbf{r}^2/2},}$ $|\psi(\mathbf{r},t)|^2 \propto \begin{cases} e^{-\mathbf{r}^2/[1+(t/2)^2]} & \text{if } g = 0 \longrightarrow \text{diffusion}, \\ e^{-|\mathbf{r}|/\xi} & \text{if } g \neq 0 \longrightarrow \text{localization}. \end{cases}$




Wavepacket width:

$\Delta\mathbf{r} = \sqrt{\langle(\mathbf{r} - \langle\mathbf{r}\rangle^2)\rangle}.$



# Conclusion and outlook

Overlook:

- Existence of an exact mapping between Bose polaron and disordered system.
- New way to probe variety of disorder physics.
- Variational method: possible simple tool to examine disorder physics.

Prospects:

- Finite mass behavior: complementary method?
- Transport properties.
- Coming soon to the arXiv!

Collaborator:
Richard Schmidt.

Thanks for your attention!