# Disorder in order:
# Anderson localization in a randomless cold atom system

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# Introduction — Disorder in quantum physics

Why study disorder in quantum physics?

Disorder is at the heart of several quantum phenomena:

- Electronic transport at low temperatures.
- Anomalous magneto-resistance.

  [Sharvin & Sharvin, JETP Lett. ’81; Pannetier et al., PRL ’84]

- Scattering of light by disorder (speckle pattern, back-scattering).

  [Kuga & Ishimaru, JOSA A ’81; Wolf & Maret, PRL ’85, Albada & Lagendijk, PRL ’85]

- Anderson localization via wave interference effects [Anderson, PR ’58].

# Introduction — Handwavey explanation of disorder effect

Question: what is the probability $n(\mathbf{r}, t)$ to diffuse from the origin to $\mathbf{r}$ in a time $t$?

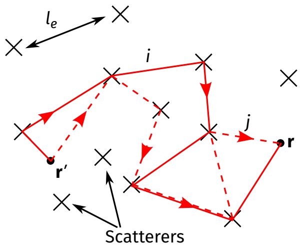
"Simple" image: $n(\mathbf{r}, t)$ expressed as a sum over scattering paths.

$$n(\mathbf{r}, t) = \Big| \sum_{\text{path } i} A_i \Big|^2 = \underbrace{\sum_{\text{path } i} |A_i|^2}_{\text{classical}} + \underbrace{\sum_{\text{path } i \neq \text{path } j} A_i A_j^*}_{\text{quantum}}.$$

Classical contribution: diffusion.

Quantum corrections (path interference) that survive disorder average can reduce diffusion.

→ localization.




$l_e$: elastic mean-free path.

1 and 2$d$: always localized, 3$d$: Anderson transition.

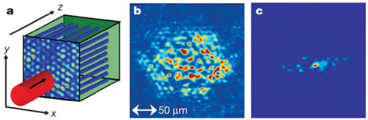
Image only valid in the weak-disorder regime, $kl_e \gg 1$.

# Introduction — Observations of Anderson localization

Initial motivation: spin transport in doped semiconductors [Feher and Gere, PR '59].
Issue: difficult to separate disorder and many-body effects...

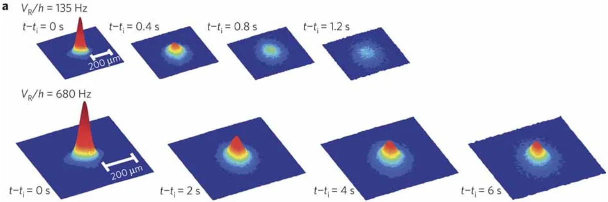
Wave phenomenon: observation with

- microwaves [Dalichaouch et al., Nature '91];
- light waves [Schwartz et al., Nature '07].




Light propagating through a photonic lattice without (left) and with (right) disorder.

Experiments in atomic matter waves [Billy et al., Nature '08; Jendrzejewski et al., Nature '12].

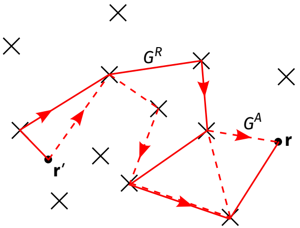



Expansion of a 3*d* BEC in presence of a weakly- (top) or strongly- (top) disordered speckle potential.

# Introduction — perturbation theory

More rigorous treatment: perturbation theory in powers of the disorder potential!

[Akkermans and Montambaux, CUP '07]




Scattering event ≃ vertex

$A_i, A_i^* \simeq G^{R,A}$

## Probability of quantum diffusion

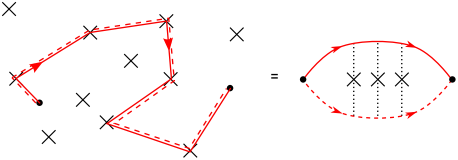
$$n(\mathbf{r},\omega) \propto \overline{G_E^R(0,\mathbf{r})G_{E-\omega}^A(\mathbf{r},0)} \quad (E\text{: particle energy})$$

$G_E^{R,A}(0,\mathbf{r}) = \langle 0|[\hat{H} - (E \pm i0^+)]^{-1}|\mathbf{r}\rangle$: Green's functions.

($\overline{\bullet}$: disorder average)

Path = diagram
Dephasing = relevance of the path
Family of paths = class of diagrams

E.g.: equal paths = ladder diagrams
→ (classical) diffusion

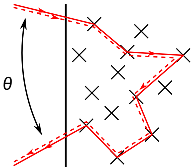


# Introduction — Coherent back-scattering

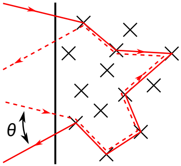
Time-reversal symmetry → reversed path has no dephasing...

...provided startpoint = endpoint!

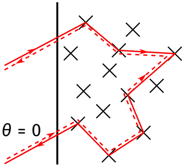
Exemple: light shined at a colloid → coherent backscattering.



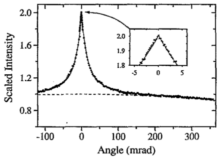

Diffusion




Coherent, dephased

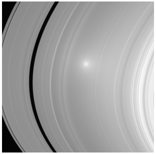



Coherent backscattering




[Wiersma et al., RSI ’95]

CBS in $BaSO_4$.



(NASA)

Saturn rings.

White spot → CBS.

# Introduction — A localization effect?

Backscattering: evidence of weak localization.

- Similar effect in electron transport: anomalous magnetoresistance.
  Magnetic field breaks down TRS → dephasing, enhanced conductivity.
- ...but nonetheless perturbative effect: in $3d$, Anderson localization happens at strong disorder.
- Other methods to go beyond: replica trick, supersymmetry.

**Main message**

- Even for non-interacting problems, disorder is complicated.
- Methods connected to many-body physics!

# Introduction — Contents

Our goals:

- Set up a mapping between a disordered problem and a polaron many-body system.
- Extend of the mapping, experimental relevance?
- Comparison of exact results for disordered problem and variational methods for the polaron.

# A simple model of disorder

Edwards model: describe e.g. magnetic impurities in a metal, [Edwards, PM '58]

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + \underbrace{V(\hat{\mathbf{r}})}_{\text{random potential}}, \qquad V(\hat{\mathbf{r}}) = \sum_{i=1}^{N} v(\hat{\mathbf{r}} - \mathbf{r}_i).$$

- $\mathbf{r}_i$: position of $N$ scattering impurities, chosen randomly (e.g. uniform).
- $v(\mathbf{r})$: scattering potential, e.g. $v(\mathbf{r}) = g\delta(\mathbf{r})$ (random Kronig-Penney model).

## Disorder average of observables

$$\overline{\langle \hat{n}(\mathbf{r}, t) \rangle} \propto \int \mathrm{d}\mathbf{r}_1 \cdots \mathrm{d}\mathbf{r}_N \, \langle \psi^{\{\mathbf{r}_i\}}(t) | \hat{n}(\mathbf{r}) | \psi^{\{\mathbf{r}_i\}}(t) \rangle,$$

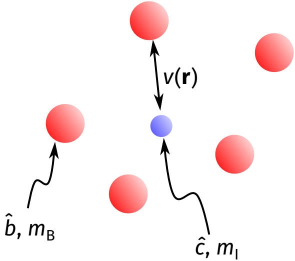
$|\psi^{\{\mathbf{r}_i\}}(t)\rangle = e^{-i\hat{H}^{\{\mathbf{r}_i\}}t} |\psi_0\rangle$ : state evolved for given disorder configuration $\{\mathbf{r}_i\}$.

# Bose polaron model

## Fermionic impurity immersed in a bosonic bath

$$\hat{H} = \underbrace{\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}} \hat{c}_{\mathbf{k}}}_{\hat{c}:\ \text{impurity (free particle)}} + \underbrace{\sum_{\mathbf{k}} \omega_{\mathbf{k}} \hat{b}^\dagger_{\mathbf{k}} \hat{b}_{\mathbf{k}}}_{\hat{b}:\ \text{bath with no interactions}} + \underbrace{\int_{\mathbf{r},\mathbf{r}'} \hat{c}^\dagger_{\mathbf{r}} \hat{c}_{\mathbf{r}} v(\mathbf{r} - \mathbf{r}') \hat{b}^\dagger_{\mathbf{r}'} \hat{b}_{\mathbf{r}'}}_{\text{interspecies interaction}} .$$

- $\hat{c}$: single impurity → wavefunction $\psi(\mathbf{r})$.
- $\hat{b}$: free boson bath, prepared initially in $|\mathrm{BEC}\rangle \propto (\hat{b}^\dagger_{\mathbf{k}=0})^N |0\rangle$.
- $v(\mathbf{r})$: density-density interaction.




Recent observations:

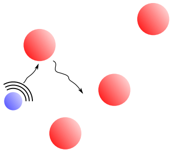
$^{39}$K in $^{39}$K [Jørgensen et al., PRL '16], $^{40}$K in $^{87}$Rb [Hu et al., PRL '16], $^{40}$K in $^{23}$Na [Yu et al., arXiv '19].

Heavy bath limit: $m_\mathrm{B} \to \infty, \omega_\mathbf{k} \to 0$.

**Step 1**: assume that at $t = 0$, $|\Xi_0\rangle = |\psi\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle$.

[Grover and Fischer, JSM '14]

Then $\begin{cases} m_\mathrm{B} = \infty \to \text{bosons remain in } |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle, \\ \text{impurity feels potential of scatterers at } \{\mathbf{r}_i\}. \end{cases}$



Massive bosons

≃ fixed scatterers.

i.e. $|\Xi(t)\rangle = |\psi^{\{\mathbf{r}_i\}}(t)\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle$

where $|\psi^{\{\mathbf{r}_i\}}(t)\rangle$ is evolved with $\underbrace{H^{\{\mathbf{r}_i\}} = \frac{\hat{\mathbf{p}}^2}{2m_\mathrm{I}} + \sum_{i=1}^{N} v(\hat{\mathbf{r}} - \mathbf{r}_i).}_{\text{Edwards Hamiltonian}}$

# Mapping to the Edwards model II

(Because $|\mathrm{BEC}\rangle \propto (\hat{b}^\dagger_{\mathbf{k}=0})^N|0\rangle$!)

**Step 2**: if system initially prepared in $|\Xi_0\rangle = |\psi_0\rangle \otimes |\mathrm{BEC}\rangle \propto \int_{\{\mathbf{r}_i\}} |\psi\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle$,

$$|\Xi(t)\rangle \propto \int_{\{\mathbf{r}_i\}} |\psi^{\{\mathbf{r}_i\}}(t)\rangle \otimes |\mathbf{r}_1, \dots, \mathbf{r}_N\rangle.$$

For any observable $\hat{O}$ of the impurity (e.g. $\hat{\mathbf{r}}$, $\hat{\mathbf{r}}^2$, $\hat{n}(\mathbf{r})$),

$$\boxed{\underbrace{\langle \Xi(t)|\hat{O}|\Xi(t)\rangle}_{\text{many-body}} = \int_{\{\mathbf{r}_i\}} \underbrace{\langle \psi^{\{\mathbf{r}_i\}}(t)|\hat{O}|\psi^{\{\mathbf{r}_i\}}(t)\rangle}_{\text{single particle}}.}$$

Many-body measurement ≡ disorder average for the Edwards model.

# Properties of the mapping

Disorder-free, many-body bose polaron model

$\Longleftrightarrow$

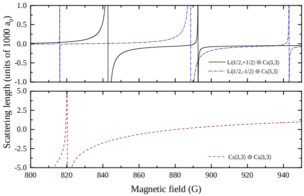
disordered, single-particle Edwards model.

Generality of the mapping?

- $v(\mathbf{r})$ can be any potential $\rightarrow$ probe universality of disorder.
- Bath prepared in state $|\Phi\rangle \rightarrow \{\mathbf{r}_i\}$ sampled with $p(\{\mathbf{r}_i\}) = |\langle\{\mathbf{r}_i\}|\Phi\rangle|^2$.
  More complicated disorders can be explored: $|\Phi\rangle$ Fermi sea?
- Remains valid with several impurities: metallic transport, many-body localization?

Limit: in real life $m_\mathrm{B} < \infty$. Hopefully not too bad at short times.

# Leads for observation?




Candidate for large mass imbalance: [Häfner et al., PRA '17]

$^6$Li in $^{133}$Cs, $m_B/m_l = 22.1.$

Cs-Li $B \simeq 889$G resonance, $a_{\text{Cs-Cs}} = 150a_0$.

**Could 3$d$ Anderson localization be investigated?**

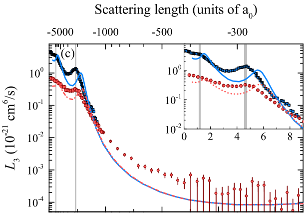
From disordered metals: transition at $k_F l_e \sim 1$; mean free path $l_e \simeq 1/(n_{\text{Cs}} a^2_{\text{Li-Cs}})$.

Three body losses: $\partial_t N_{\text{Li}}/N_{\text{Li}} \sim -L_3(a_{\text{Li-Cs}}) n^2_{\text{Cs}}$.

Possible observation: $a_{\text{Li-Cs}}$ "small" and

- Fermi sea of $^6$Li;
- few impurities at "small" **k**.

  E.g.: Raman spectroscopy, [Shkedrov et al., arXiv '19].



# Application — estimating the variational method

Possible application of the mapping: compare, for $1d$, the exact results for the Edwards model and approximate variational method for the polaron model

Scenario: impurity prepared in gaussian wavepacket $\psi(\mathbf{r}, t = 0) \propto e^{-\mathbf{r}^2/2\sigma^2}$.

$$\boxed{\psi(\mathbf{r}, t) = ?}$$

Interest:

- Showcase the mapping!
- Benchmark the variational *Ansätze*.
- Stepping stone towards variational method where there is no exact solution (higher dimension, finite boson mass).

# Exact solution of the Edwards model

**1d Hamiltonian**

$$\hat{H} = \frac{-\partial_x^2}{2m} + g\sum_{i=1}^{N} \delta(x - x_i)$$

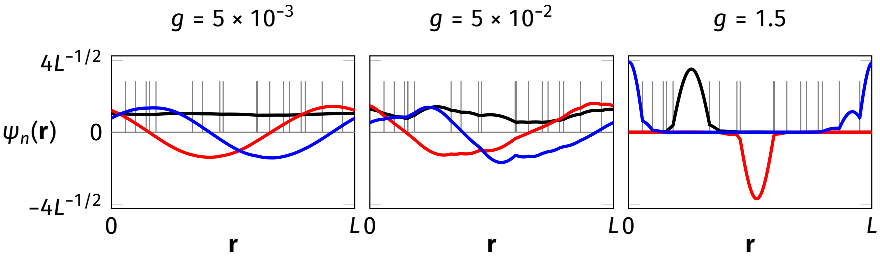
$\hat{H}\psi = E\psi$ with $E = k^2/2m$ implies

- for $x_i < x < x_{i+1}$, $\psi(x) = A_i \sin(kx + \varphi_i)$;
- at $x_i$, $\psi'(x_i^+) - \psi'(x_i^-) = 2mg\psi(x_i)$.

System of equations for $A_i$, $\varphi_i$.

[Nieuwenhuizen, Physica A ’83]

- Boundary conditions → spectrum.
- Numerical solution of the system → eigenstates!




First three eigenstates $\psi_1$, $\psi_2$, $\psi_3$ for different disorder strengths at given $x_i$.

# Variational principle — method

**Polaron Hamiltonian**

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} + \sum_{\mathbf{k}} \omega_{\mathbf{k}} \hat{b}^{\dagger}_{\mathbf{k}} \hat{b}_{\mathbf{k}} + \int_{\mathbf{r},\mathbf{r}'} \hat{c}^{\dagger}_{\mathbf{r}} \hat{c}_{\mathbf{r}} v(\mathbf{r} - \mathbf{r}') \hat{b}^{\dagger}_{\mathbf{r}'} \hat{b}_{\mathbf{r}'}$$

Our goal: simple study of the polaron model via time-dependent variational method.

(see e.g Tommaso's talk!)

Idea: restrict oneself to a variationnal manifold $\mathcal{M} = \{|\Xi(z_i)\rangle, z_i \in \mathbb{C}\}$.

Schrödinger equation $i\partial_t|\Xi\rangle = \hat{H}|\Xi\rangle$ is approximated by

- minimizing $\|(i\partial_t - \hat{H})|\Xi\rangle\|$ wrt $\dot{z}_i$;
- eqs. of motion of Lagrangian $L = \langle\Xi|i\partial_t - \hat{H}|\Xi\rangle$ wrt $z_i$, $\dot{z}_i$;
- projecting out $i\partial_t|\Xi\rangle = \hat{H}|\Xi\rangle$ on $\mathcal{M}$.

Equivalent under reasonable assumptions! ($\partial_{z_i^*}|\Xi\rangle = 0$)

# Variational method — Chevy *Ansatz*

Simplest polaron *Ansatz* [Chevy, PRA '06]:

$$|\Xi(t)\rangle = \alpha_0 \underbrace{|\mathbf{p}\rangle \otimes |\mathrm{BEC}\rangle}_{\text{initial state}} + \sum_{\mathbf{q}\neq 0} \alpha_{\mathbf{q}} \underbrace{|\mathbf{p}+\mathbf{q}\rangle \otimes \hat{b}^\dagger_{-\mathbf{q}} \hat{b}_0 |\mathrm{BEC}\rangle}_{\text{single excitation of the BEC}}$$

- Only allow for a single excitation of the BEC.
- Wavepacket reconstructed by adding different modes.

Issue: we except that the impurity should exchange small momentum with many bosons → not covered by the *Ansatz*...

# Variational method — Coherent state

Tool: Lee–Low–Pines transform [Lee et al., PR '53].

Idea: move to the reference frame of the impurity via $\hat{S} = \exp\Big(i\overbrace{\hat{\mathbf{r}}}^{\text{impurity}}\cdot\underbrace{\sum_{\mathbf{k}}\mathbf{k}\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}}}_{\text{boson momentum}}\Big)$.

In the new frame $\hat{H}^{\text{LLP}} = \hat{S}^\dagger\hat{H}\hat{S}$, **the impurity momentum $\mathbf{p}$ is conserved.**

Extensively used: [Devreese and Alexandrov, RPP '09; Shashi et al., PRA '14; Schadilova et al., PRL '16...].

$|\Xi_0\rangle = |\mathbf{p}\rangle \otimes |\text{BEC}\rangle \Longrightarrow |\Xi(t)\rangle = |\mathbf{p}\rangle \otimes |\text{BEC}_{\mathbf{p}}(t)\rangle$,

the BEC is evolved under a $\mathbf{p}$-dependent Hamiltonian $\hat{H}^{\mathbf{p}}$.

$$\hat{H}^{\mathbf{p}} = \frac{(\mathbf{p} - \sum_{\mathbf{k}}\mathbf{k}\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}})^2}{2m_I} + \sum_{\mathbf{k}}\omega_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}} + g\sum_{\mathbf{k}\mathbf{k}'}\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}'}.$$

Coherent state approximation: $|\text{BEC}_{\mathbf{p}}(t)\rangle \propto \exp\Big(\sum_{\mathbf{k}}\beta^{\mathbf{p}}_{\mathbf{k}}(t)\hat{b}^\dagger_{\mathbf{k}}\Big)|0\rangle$.

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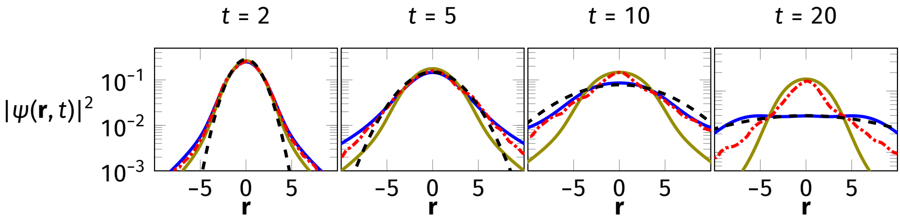
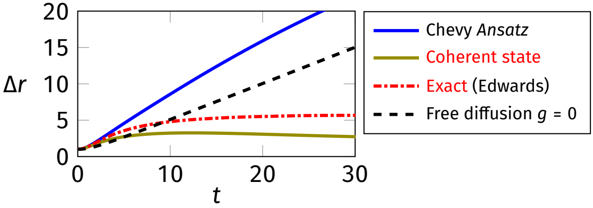
# Results: spread of the wavepacket

Localization is observed on the spread of a wavepacket: $[\mathbf{r} \to \mathbf{r}/\sigma, t \to t/(2m_I\sigma^2/\hbar)]$

If $\boxed{\psi(\mathbf{r}, t=0) \propto e^{-\mathbf{r}^2/2}}$, $|\psi(\mathbf{r},t)|^2 \propto \begin{cases} e^{-\mathbf{r}^2/[1+(t/2)^2]} & \text{if } g = 0 \longrightarrow \text{diffusion, } \Delta r \sim t/2 \\ e^{-|\mathbf{r}|/\xi} & \text{if } g \neq 0 \longrightarrow \text{localization, } \Delta r \sim \xi. \end{cases}$




Wavepacket width:
$$\Delta r = \sqrt{\langle(\mathbf{r} - \langle\mathbf{r}\rangle^2)\rangle}.$$



# Conclusion and outlook

Overlook:

- Existence of an exact mapping between Bose polaron and disordered system.
- New way to probe variety of disorder physics.
- Variational method: possible simple tool to examine disorder physics.
- Chevy misses localization, coherent states capture qualitative behavior.

Prospects:

- Gaussian states?
- Finite mass behavior: complementary method?
- Transport properties.
- Coming soon to the arXiv!

Collaborator:
Richard Schmidt.

Thanks for your attention!