Higgs mode and conductivity in the vicinity of a quantum critical point

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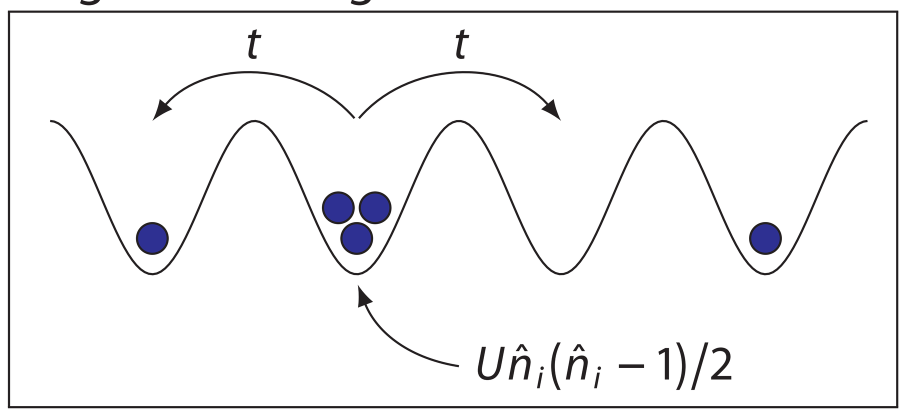
Quantum phase transitions and the O(N) model

Quantum phase transition (QPT):
a qualitative change in the ground stade as an external parameter is varied.

Eg: Interacting lattice bosons.



$$\hat{H} = t\sum_{\langle i,j\rangle}\left(\hat{b}_i^\dagger \hat{b}_j + \text{h.c.}\right) + U\sum_i \frac{\hat{n}_i(\hat{n}_i-1)}{2} - \mu\sum_i \hat{n}_i$$

Phase diagram (integer filling):

$(t/U)_c$ t/U

Insulating phase (localized bosons) | Superfluid phase (condensate)

Path integral formulation: partition function $\mathcal{Z} = \int \mathcal{D}[\boldsymbol{\varphi}]\exp(-S[\boldsymbol{\varphi}])$. O(N) universality class:

$$\text{action } S[\boldsymbol{\varphi}] = \int_0^\beta d\tau \int d^d\mathbf{r} \left\{ \frac{1}{2}(\partial_i\boldsymbol{\varphi})^2 + \frac{1}{2c^2}(\partial_\tau\boldsymbol{\varphi})^2 + \frac{r_0}{2}\boldsymbol{\varphi}^2 + \frac{u_0}{4!}(\boldsymbol{\varphi}^2)^2 \right\}.$$

Physical realizations: $N=2$ describes SF-MI transition, $N=3$ antiferromagnets.

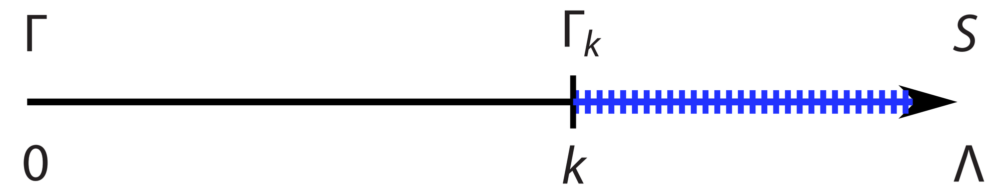
$\boldsymbol{\varphi}$: bosonic N-component field, $\boldsymbol{\varphi}(\mathbf{r},\tau+\beta) = \boldsymbol{\varphi}(\mathbf{r},\tau)$. UV cutoff Λ.

$T=0$ limit: change of variables $\mathbf{x} = (\mathbf{r}, c\tau)$

Non-perturbative renormalization group

NPRG: implemented on Gibbs free energy $\Gamma[\boldsymbol{\varphi}]$.
Γ: Legendre transform of free energy $\ln\mathcal{Z}$.

Idea: construct a family of theories indexed by a scale $0 \le k \le \Lambda$ such that fluctuations for scales $\lesssim k$ are frozen to interpolate between the mean-field ($k=\Lambda$) and the exact solution ($k=0$).



This is done by adding to the action a mass-like term

$$\Delta S_k[\boldsymbol{\varphi}] = \frac{1}{2}\int_{\mathbf{q}} \boldsymbol{\varphi}(\mathbf{q})\cdot R_k(\mathbf{q})\boldsymbol{\varphi}(\mathbf{q}), \qquad R_k(\mathbf{q})\text{: regulator.}$$

$R_k(\mathbf{q}) \sim k^2$ if $\mathbf{q} \lesssim k$ and ~ 0 otherwise.

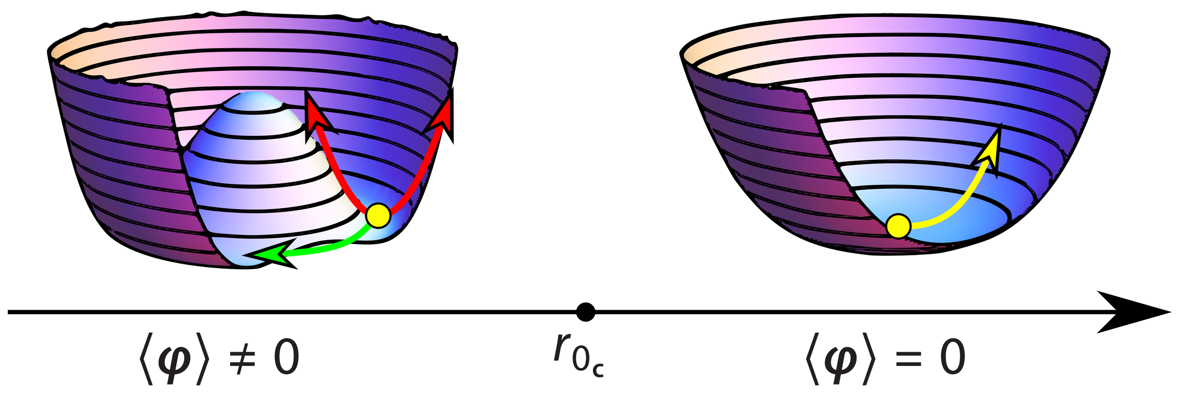
Exact flow equation: $\partial_k\Gamma_k[\boldsymbol{\varphi}] = \frac{1}{2}\text{Tr}\left\{\partial_k R_k\left(\Gamma_k^{(2)}[\boldsymbol{\varphi}] + R_k\right)^{-1}\right\}.$

Eg. of approximation scheme: the derivative expansion (DE)

$$\Gamma_k[\boldsymbol{\varphi}] = \int_{\mathbf{x}} \frac{Z_k(\boldsymbol{\varphi}^2)}{2}(\partial_\mu\boldsymbol{\varphi})^2 + U_k(\boldsymbol{\varphi}^2) + \frac{Y_k(\boldsymbol{\varphi}^2)}{4}(\boldsymbol{\varphi}\cdot\partial_\mu\boldsymbol{\varphi})^2.$$

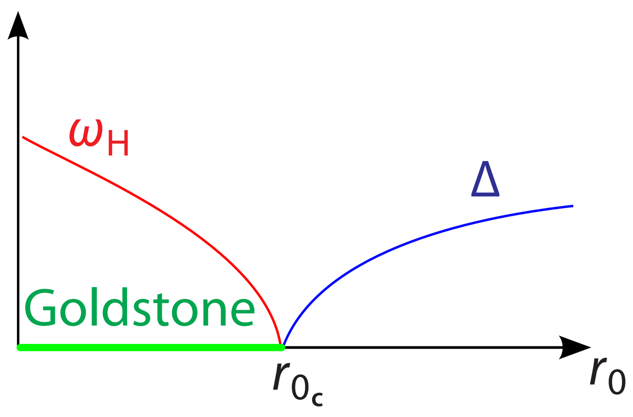
Higgs amplitude mode

Mean-field zero T phase diagram:



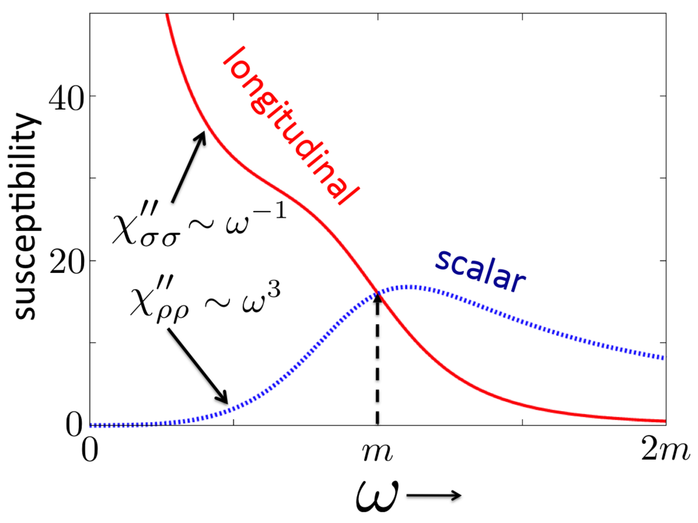
$N-1$ Goldstone modes + amplitude mode (gap ω_H).

N gapped modes (gap Δ).



Mean-field:
$\Delta = A\left|r_0 - r_{0_c}\right|^{1/2}$
$\omega_H = A\sqrt{2}\left|r_0 - r_{0_c}\right|^{1/2}$

In $d+1 = (2+1)$ MF is qualitatively wrong $\Rightarrow$ NPRG.



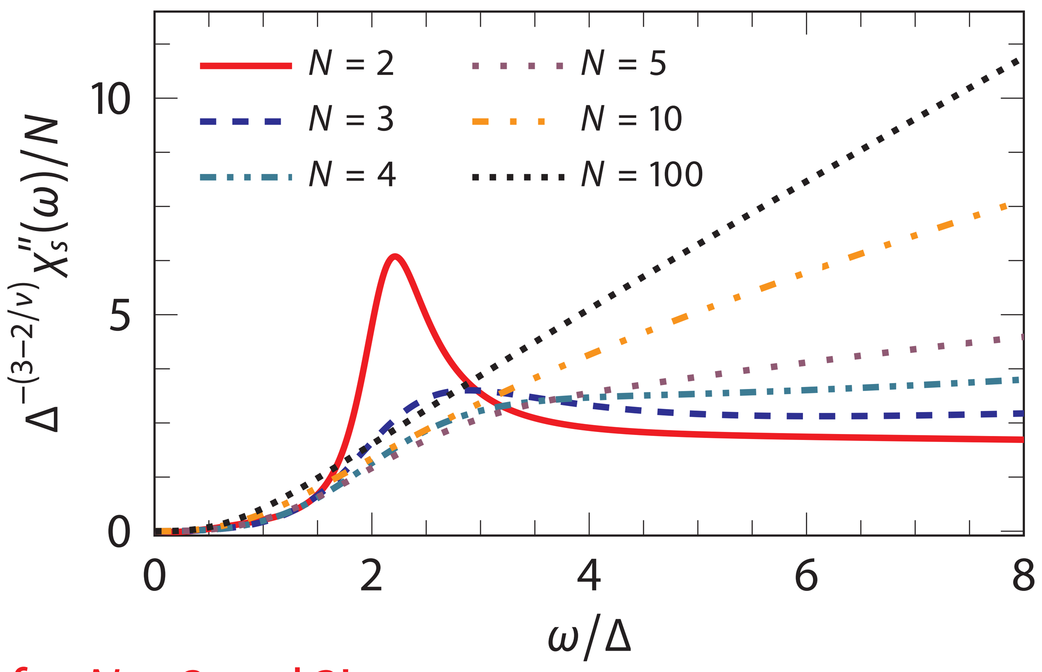
Object to compute: the scalar susceptibility [1]

$$\chi_s(\mathbf{r},\tau) = \left\langle \boldsymbol{\varphi}^2(\mathbf{r},\tau)\boldsymbol{\varphi}^2(0,0)\right\rangle,$$
$$\chi''_s(\omega) = \text{Im}[\chi_s(\mathbf{q}=0, i\omega_n \to \omega + i0^+)].$$

Scalar susceptibility computations

Our results [2] for $\chi''_s(\omega)$:

Technique:
Add source term $S \to S + \int_{\mathbf{x}} h(\mathbf{x})\boldsymbol{\varphi}(\mathbf{x})^2$.
Then $\chi_s \sim \delta^2\Gamma/\delta^2 h$. The BMW approximation allows to compute the full momentum dependence of the vertices.



Evidence of the existence of the Higgs mode for $N=2$ and 3!
Agreement with previous QMC studies [3, 4].

Conductivity

Noether: $O(N)$ symmetry $\Rightarrow$ conserved current $j^a_\mu = \boldsymbol{\varphi}\cdot T^a\partial_\mu\boldsymbol{\varphi}$. Bosons: $\mathbf{j} \sim i(\varphi^*\nabla\varphi - \varphi\nabla\varphi^*)$.
T^a: skew-symmetric matrix, $N(N-1)/2$ independent currents.

$$\text{Conductivity: } \sigma^{ab}_{\mu\nu}(i\omega_n) = -\frac{1}{\omega_n}\left[\left\langle j^a_\mu(\mathbf{q}=0,i\omega_n)j^b_\nu(\mathbf{q}=0,-i\omega_n)\right\rangle - \delta_{\mu\nu}\left\langle T^a\boldsymbol{\varphi}\cdot T^b\boldsymbol{\varphi}\right\rangle\right].$$

Symmetry and Ward identities determine its form in the low frequency limit.

- In the disordered phase there is only one independent conductivity behaving as a capacitance, $\sigma(\omega) = -i\omega C_{\text{dis}}$.
- In the ordered phase, the order parameter $\boldsymbol{\varphi}$ is finite.
  There are two independent conductivites depending on whether T^a acts on $\boldsymbol{\varphi}$ (class A) or not (class B). σ_A behaves like a perfect inductance $\sigma_A(\omega) = iL_{\text{ord}}/(\omega + i0^+)$ and σ_B has a universal finite limit.
- At criticality σ^* reaches a universal finite value.

Results: the ratio $C_{\text{dis}}/L_{\text{ord}}$ is universal!
For $N=2$: $C_{\text{dis}}/L_{\text{ord}} = 0.105(q^2/h)^2$ [5], in agreement with QMC [6].

Technique: introduce a source gauge field $\partial_\mu\boldsymbol{\varphi} \to (\partial_\mu - A_\mu)\boldsymbol{\varphi}$.
BMW breaks Ward identities $\Rightarrow$ we make a derivative expansion of the effective action in powers of ∂_μ and A_μ.
Then $\sigma \propto \delta^2\Gamma/\delta A^2$ is derived at low frequencies.

Advantages of NPRG over other standard techniques to compute transport quantities:

- QMC: no data noise issues means smoother analytic continuation.
- AdS/CFT: link with condensed matter models easier to derive.

References

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